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Bifurcation Analysis and Dimension Reduction of a Predator-Prey Model for the L-H Transition

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The L-H transition denotes a shift to an improved confinement state of a toroidal plasma in a fusion reactor. A model of the L-H transition is required to simulate the time dependence of tokamak discharges that include the L-H transition. A 3-ODE predator-prey type model of the L-H transition is investigated with bifurcation theory of dynamical systems. The model is recognized as a slow-fast system.

INTRODUCTION

The L- and H-modes¹ are confinement states of a toroidal plasma, referring to states of low and high confinement, respectively. The transition from the L- to the H-mode is called the L-H transition. The T-mode is a transient, intermediate mode between the L- and H-mode, characterized by an oscillatory behavior. The L-H transition still lacks a first principle explanation. Modeling the L-H transition might contribute to a better understanding of the underlying mechanisms.

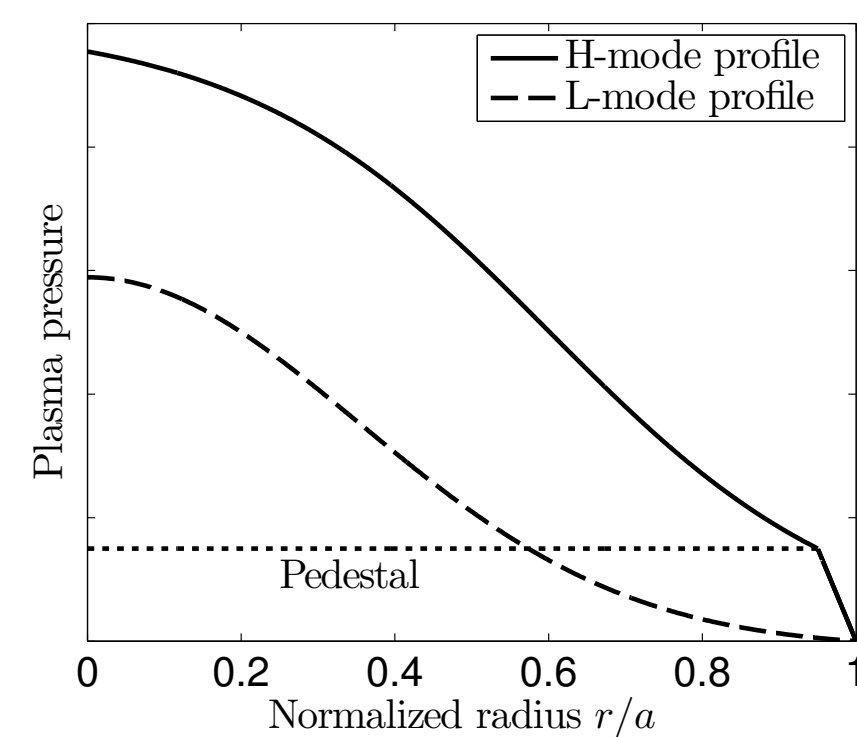


Figure: Sketch of the radial pressure profiles in L- and H-mode.

THE 3-ODE L-H TRANSITION MODEL

We consider the minimal 3-ODE predator-prey type L-H transition model suggested by Kim and Diamond [2, 3]. The model ignores spatial dependencies. The dependent variables in the model are:

- ▶ the drift wave turbulence level \mathcal{E} ,
- ▶ the shear of the zonal flow V_{zf} , and
- ▶ the gradient of the ion pressure \mathcal{N} .

The model can be formulated as

$$\begin{aligned}\frac{d}{dt}\mathcal{E} &= \mathcal{E} (\mathcal{N} - a_1\mathcal{E} - a_2c_3^2\mathcal{N}^4 - a_3V_{zf}^2), \\ \frac{d}{dt}V_{zf} &= V_{zf} \left(\frac{b_1\mathcal{E}}{1 + b_2c_3^2\mathcal{N}^4} - b_3 \right), \\ \frac{d}{dt}\mathcal{N} &= Q(t) - \mathcal{N}(c_1\mathcal{E} + c_2),\end{aligned}$$

where $a_i, b_i, c_i, i = 1, 2, 3$ are parameters and Q is the heating power.

Introducing new variables and time,

$$\begin{aligned}u &= a_1a_2^{1/3}c_3^{2/3}\mathcal{E}, & v &= a_2^{1/3}a_3c_3^{2/3}V_{zf}^2, \\ w &= a_2^{1/3}c_3^{2/3}\mathcal{N}, & s &= a_2^{-1/3}c_3^{-2/3}t,\end{aligned}$$

results in the non-dimensionalized system

$$\begin{aligned}\dot{u} &= u(w - u - v - w^4) \\ \dot{v} &= \mu_1 v \left(\frac{u}{1 + \mu_4 w^4} - \mu_2 \right) \\ \dot{w} &= \mu_5 (\sigma - w(1 + \mu_3 u))\end{aligned}$$

Here, $\mu_i, i = 1, \dots, 5$ are new parameters and σ is the rescaled heating power.

BIFURCATION ANALYSIS

The nullclines are

$$\begin{aligned}\mathcal{N}_u &= \{u = 0\} \cup \{u = w(1 - w^3) - v\}, \\ \mathcal{N}_v &= \{v = 0\} \cup \{u = \mu_2(1 + \mu_4 w^4)\}, \\ \mathcal{N}_w &= \{w(1 + \mu_3 u) = \sigma\}.\end{aligned}$$

Stability of the equilibrium points:

- ▶ L is a stable node when it is below \mathcal{N}_v .
- ▶ H is always a saddle (unstable).
- ▶ T is a focus point. Stability depends on the value of σ .
- ▶ QH is stable for $\sigma > 1$.

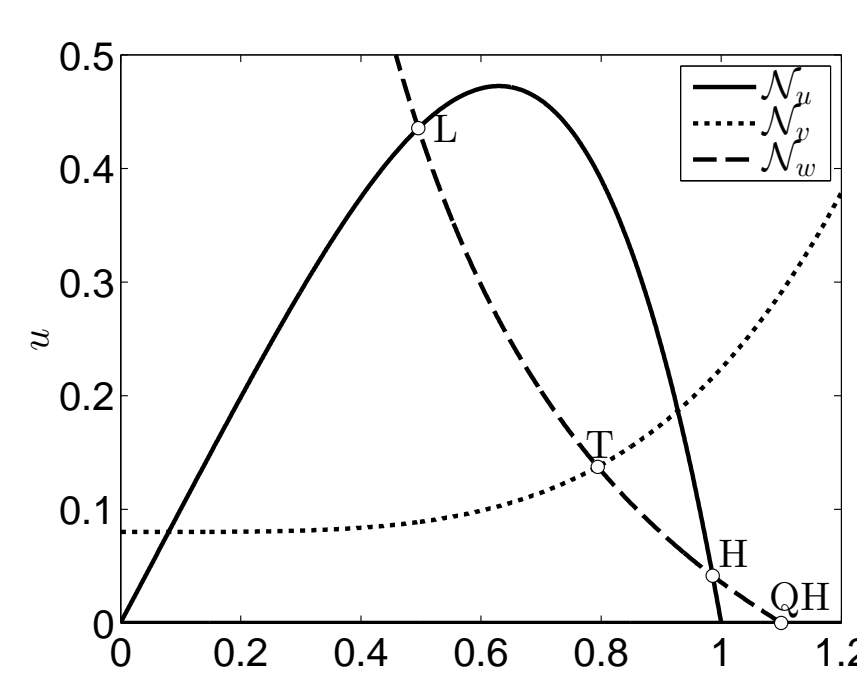


Figure: Nullclines and equilibrium points projected onto the uw -plane.

THREE TRANSITION TYPES

The bifurcation diagram structure depends on $\mu_i, i = 1, \dots, 5$. By varying μ_2 and μ_3 the three different transition types are observed.

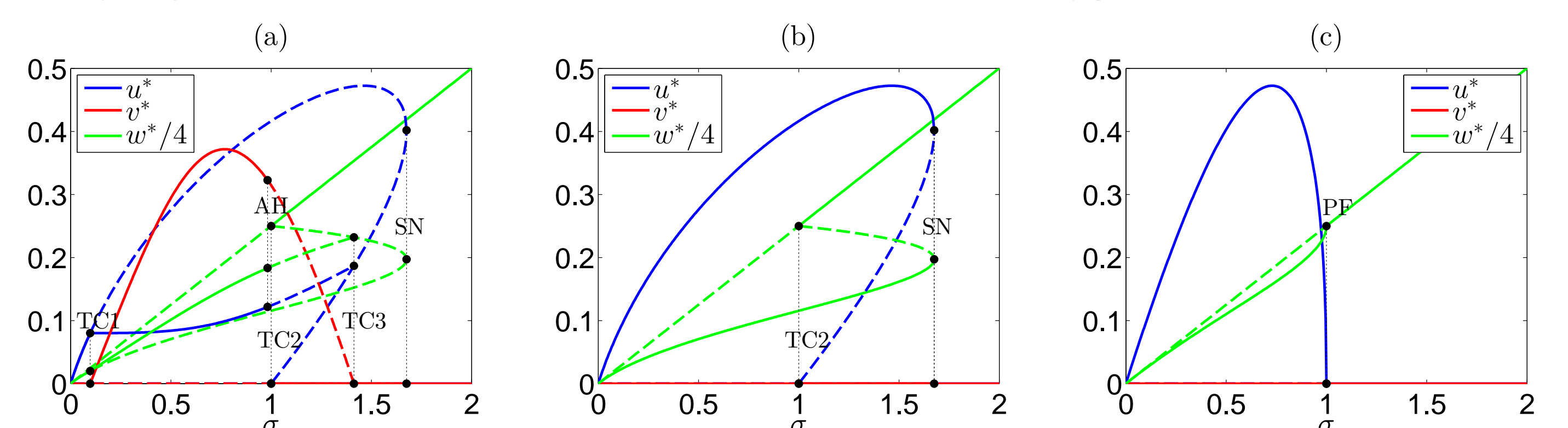


Figure: Bifurcation diagrams showing the three types of transitions: (a) an oscillating transition, (b) a sharp transition allowing hysteresis, and (c) a smooth transition. Stable equilibria are shown as solid curves and unstable equilibria as dashed curves.

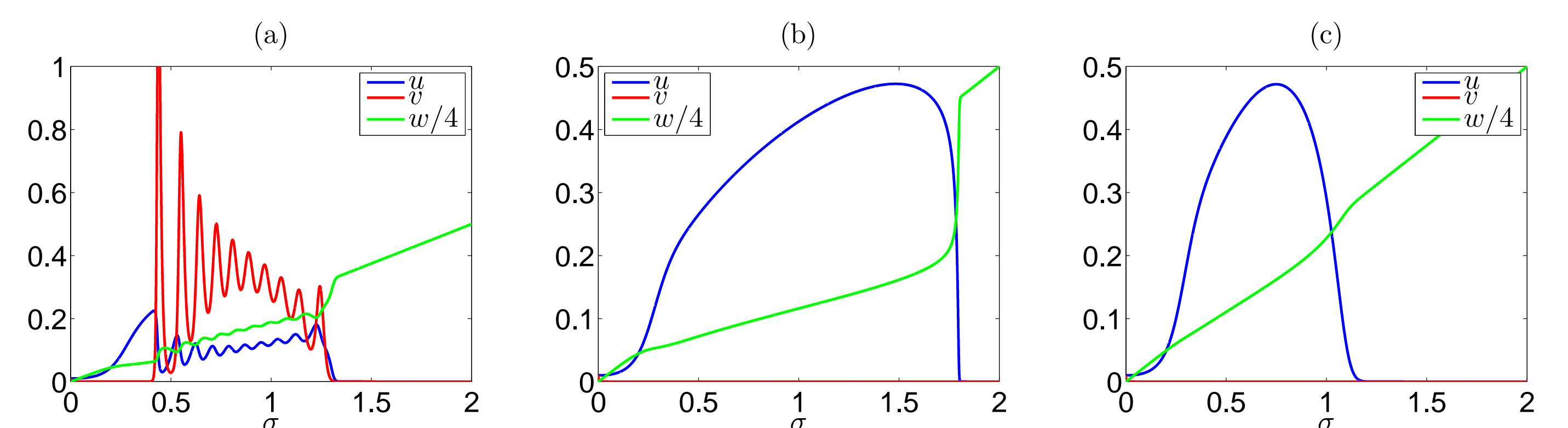


Figure: Numerical solutions showing the three types of transitions: (a) an oscillating, (b) a sharp, and (c) a smooth transition.

DIMENSION REDUCTION WITH GSPT

Put $\mu_5 = \frac{1}{\varepsilon}$, where $0 < \varepsilon \ll 1$.

- ▶ u and v are slow variables,
- ▶ w is a fast variable.

For $\varepsilon > 0$, but sufficiently small, solutions converge to the slow manifold,

$$\mathcal{M}_\varepsilon = \mathcal{M}_0 + \varepsilon \mathcal{M}_1 + \varepsilon^2 \mathcal{M}_2 + \dots$$

The reduced system of the flow is found by taking the limit $\varepsilon \rightarrow 0$:

$$\begin{aligned}\dot{u} &= u(w - u - v - w^4) \\ \dot{v} &= \mu_1 v \left(\frac{u}{1 + \mu_4 w^4} - \mu_2 \right) \\ w &= \frac{\sigma}{1 + \mu_3 u}\end{aligned}$$

The reduced system contains the same dynamics as the full system.

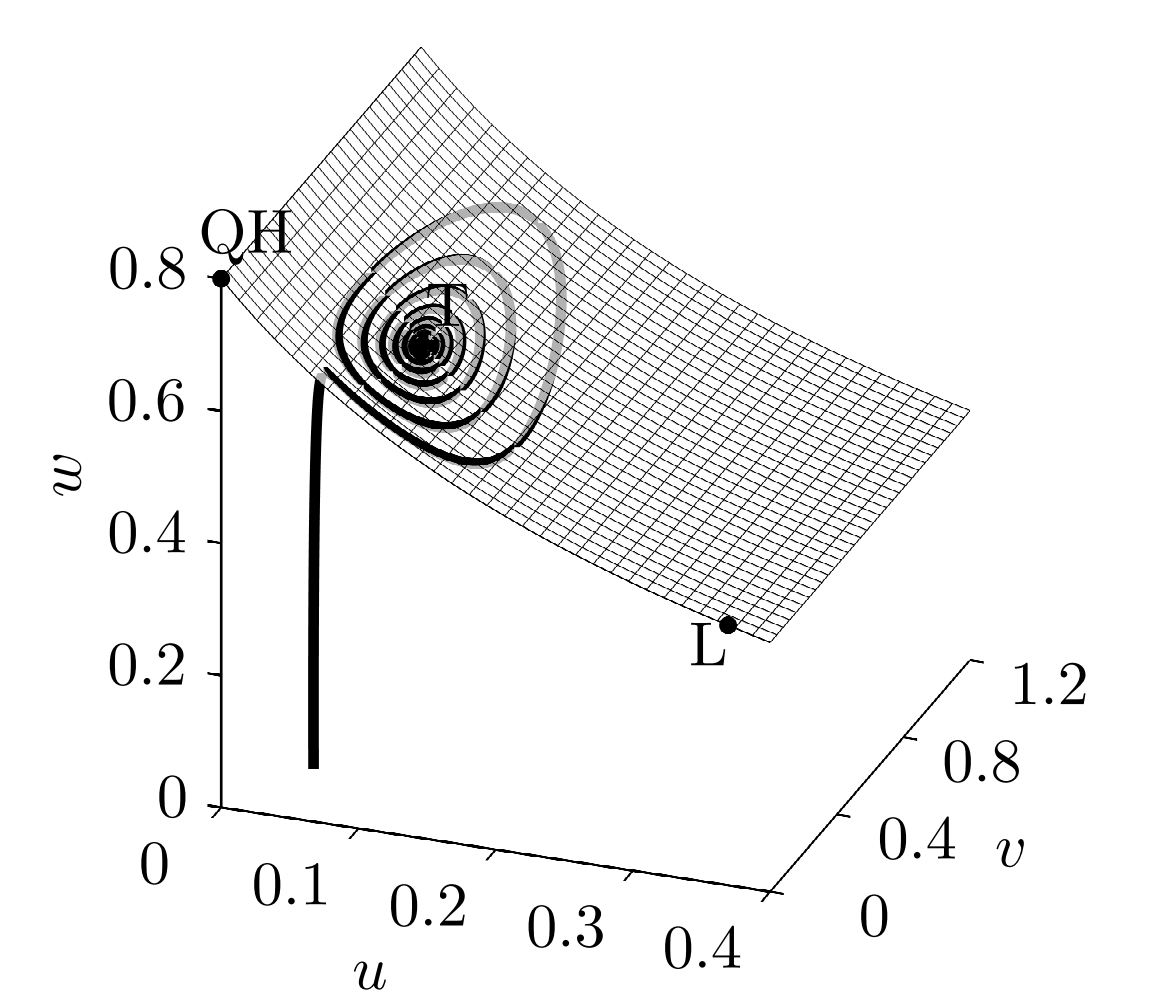


Figure: The critical manifold, $\mathcal{M}_0 = \{w = \frac{\sigma}{1 + \mu_3 u}\}$ and a numerical solution.

CONCLUSION

Kim and Diamond's 3-ODE L-H transition model was investigated with bifurcation theory.

- ▶ The model contains three types of transitions: an oscillating, a sharp with hysteresis, and a smooth transition.
- ▶ The system can be reduced to a 2-ODE system

A spatio-temporal L-H transition model has been proposed by Miki et al. [4].

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